

Laws of resistance and various types of resistors

Objectives: At the end of this lesson you shall be able to

- state the laws of resistance, compare resistances of different materials
- state the relationship between the resistance and diameter of a conductor
- calculate the resistance and diameter of a conductor from the given data (i.e. dimensions etc.)
- explain various types of resistors.

Laws of resistance: The resistance R offered by a conductor depends on the following factors.

- The resistance of the conductor varies directly with its length.
- The resistance of the conductor is inversely proportional to its cross-sectional area.
- The resistance of the conductor depends on the material with which it is made of.
- It also depends on the temperature of the conductor.

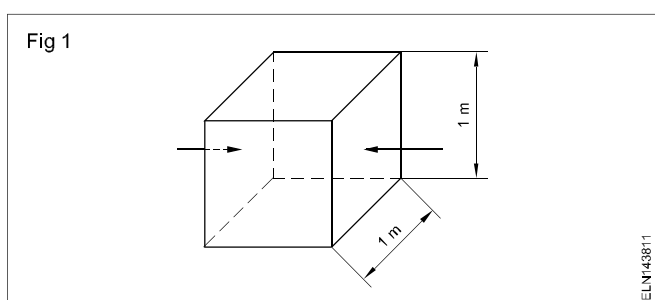
Ignoring the last factor for the time being, we can say that

$$R = \frac{L}{a}$$

where ' ρ ' (rho - Greek alphabet) - is a constant depending on the nature of the material of the conductor, and is known as its **specific resistance** or **resistivity**.

If the length is one metre and the area, ' a ' = 1 m², then $R = \rho$.

Hence, specific resistance of a material may be defined as 'the resistance between the opposite faces of a metre cube of that material'. (or, sometimes, the unit cube is taken in centimetre cube of that material) (Fig 1).



$$\text{We have } \rho = \frac{aR}{L}$$

In the SI system of units

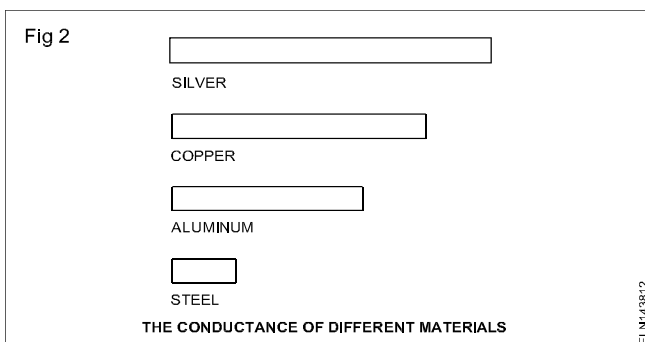
$$= \frac{a \text{ metre}^2 \times R \text{ ohm}}{L \text{ metre}}$$

$$= \frac{aR}{L} \text{ ohm - metre}$$

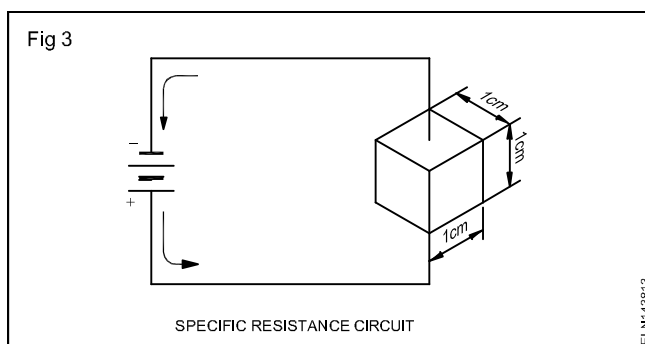
Hence the unit of specific resistance is ohm metre (Ωm).

Comparison of the resistance of different materials:

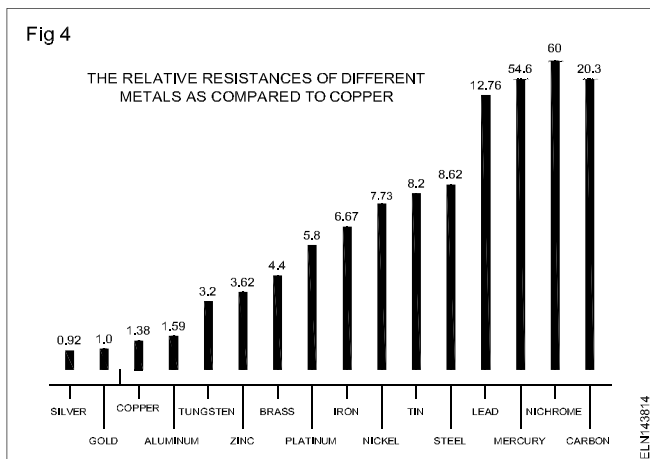
Fig 2 gives some relative idea of the more important materials as conductors of electricity. All the conductors shown have the same cross-sectional area and the same amount of resistance. The silver wire is the longest while that of copper is slightly short and that of aluminium is shorter still. The silver wire is more than 5 times longer than the steel wire.



Since different metals have different conductance ratings, they must also have different resistance ratings. The resistance ratings of the different metals can be found by experimenting with a standard piece of each metal in an electric circuit. If you cut a piece of each of the more common metals to a standard size, and then connect the pieces to a battery, one at a time, you would find that different amounts of current would flow. (Fig 3)



The bar graph (Fig 4) shows the resistance of some common metals as compared to copper. Silver is a better conductor than copper because it has less resistance. Nichrome has 60 times more resistance than copper, and copper will conduct 60 times as much current as Nichrome, if they were connected to the same battery, one at a time.



Relationship between the resistance and the diameter of a conductor: For a uniform wire of a given material, the value obtained by dividing the P.D. between any two points by the current is the resistance between those two points, and is directly proportional to the distance between them.

Also if two equal value resistors, each having resistance R , are connected in parallel it's equivalent R_T is given by

$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

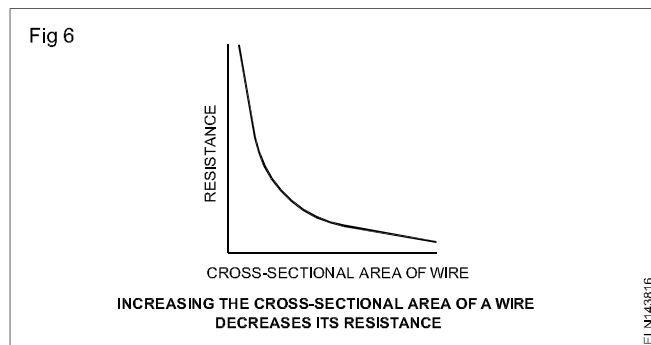
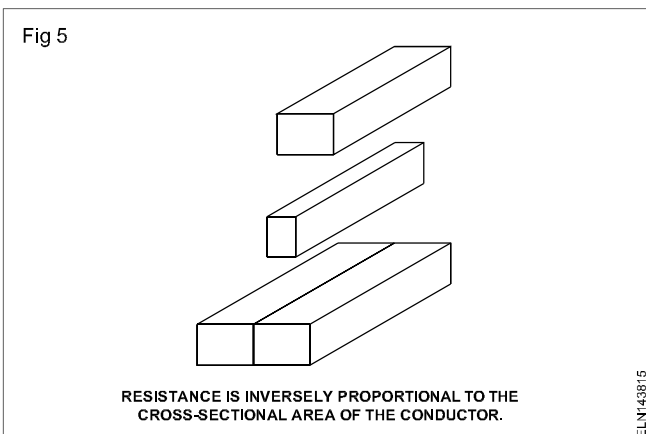
Therefore $R_T = \frac{R}{2}$

Hence, if two wires of the same material having the same length and diameter are connected in parallel the resistance of the two parallel wires is half that of one wire alone.

But the effect of connecting two wires in parallel is exactly similar to doubling the area of the conductor in just the same way as the effect of connecting, say, five wires in parallel is the same as increasing the cross-sectional area of a wire five times, and the result is to reduce the resistance to a fifth of that of one wire.

In general, we may, therefore, say that the resistance of a given length of a conductor is inversely proportional to its cross-sectional area.

The other factor that influences the resistance is the nature of the material. Hence, we may now say that resistance of a wire (Fig 5 & Fig 6)



$$= \frac{\text{length}}{\text{area}} \times (\text{a constant}) \rho \text{ given material}$$

$$R(\text{ohms}) = \frac{L(\text{metres})}{a \text{ metre}^2} \times$$

So that $\rho = Ra \div L$ ohm/ meter

where ρ (greek letter, pronounced 'rho') represents the constant.

L is the length of the wire in metres

a is the area in square metres.

Example: Calculate the length of a copper wire of 1.5 mm diameter which is to have a resistance of 0.3 ohms given that resistivity of copper is 0.017 microhm meter.

Solution

Cross-sectional area of wire

$$= (\pi/4) \times (1.5)^2 = 1.766 \text{ mm}^2$$

$$= 1.766 \times 10^{-6} \text{ m}^2$$

$$R = \frac{L}{a}$$

$$= 0.3 = \frac{0.017 \times 10^{-6} \times L}{1.766 \times 10^{-6}}$$

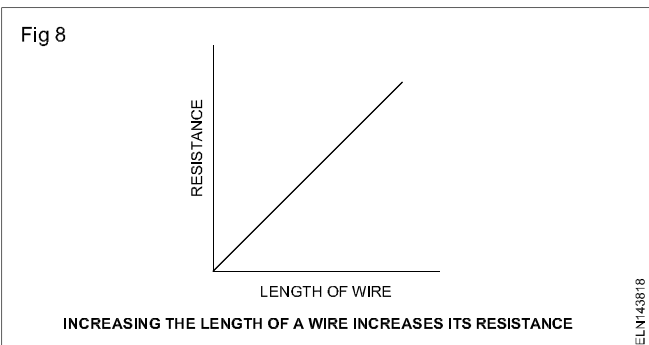
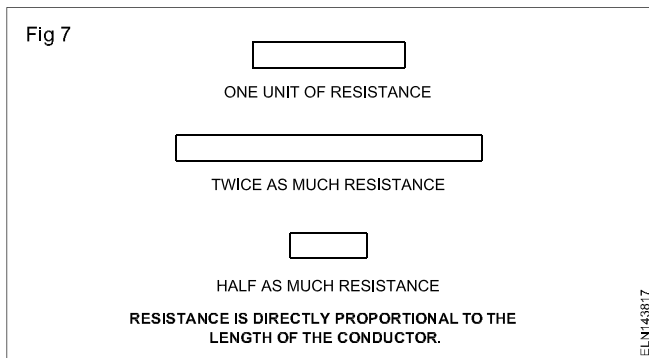
Ans: Length = 31.2 m.

We can reduce all this into a simple statement: the larger the wire, the lower its resistance; the smaller cross sectional area of the wire, the higher its resistance.

We can summarize with the universal rule: the electrical resistance of any metallic conductor is inversely proportional to its cross-sectional area.

All of this provides us with a useful rule in working with electrical conductors of any kind. Electrical resistance is directly proportional to the length of the conductor, provided, of course, the conductor is of the same diameter and is made of the same material throughout. (Figs 7 & 8)

Thus, the length of wire has a considerable influence on its ability to conduct electricity. The longer the wire, the more difficult it is for the current to get through it. In other words, the longer the wire the greater its resistance.



Calculation of resistance

Example 1: If a 15m eureka wire 0.14cm in diameter has a resistance of 3.75 ohms find the specific resistance of the material.

Solution

Length of wire $L = 15\text{m} = 15 \times 100 = 1500 \text{ cm}$

Diameter of wire = 0.14cm

Resistance = 3.75 ohm

Cross-sectional area of the wire

$$a = \pi r^2 = \frac{\pi d^2}{4}$$

we know $R = \frac{L}{a}$

$$\text{Specific resistance} = \frac{R \times a}{L}$$

$$= \frac{3.75 \times 22 \times (0.14)^2}{15 \times 100 \times 7 \times 4} \text{ ohm/cm}$$

$$= \frac{3.75 \times 22 \times (0.14)^2 \times 10^6}{15 \times 100 \times 7 \times 4} \text{ micro ohm/cm}$$

$$= 38.5 \text{ micro ohm cm}$$

$$= 38.5 \mu \text{ ohm cm.}$$

Example 2: Calculate the resistance of a 2 km long wire composed of 19 strands copper conductor, each strand being 1.32mm in diameter. Resistivity of copper may be taken as $1.72 \times 10^{-8} \text{ ohm-m}$. Allow 5% increase in length for the 'lay' (twist) of each strand in the completed cable.

Solution

Allowing for twist, the length of strands,

$$= 2000 + 5\% \text{ of } 2000 \text{ metre}$$

$$= 2100\text{m}$$

Area of cross-section of 19 strands of copper conductor is

$$= 19 \times \frac{\pi d^2}{4}$$

$$= 19 \times \pi \frac{(1.32 \times 10^{-3})^2 \text{ m}^2}{4}$$

$$\text{Now } R = \frac{\rho L}{a} = \frac{1.72 \times 10^{-8} \times 2100 \times 4 \times 7}{19 \times (1.32 \times 10^{-3})^2 \times 22}$$

$$= \frac{1.72 \times 10^{-8} \times 2100 \times 4 \times 7}{19 \times 22 \times (1.32)^2 \times 10^{-6}}$$

$$= 1.388 \text{ ohms.}$$

Example 3: Calculate in mm the dia. of a copper wire; the resistance of 3km of the wire is 14.4 ohms. Specific resistance of copper may be taken as 1.7 micro-ohm per centimetre cube.

Solution

$$\text{Length} = 3\text{km} = 3 \times 1000 \times 100$$

$$= 300\,000 \text{ cm}$$

$$\text{Resistance} = 14.4 \text{ ohms}$$

$$\rho = 1.7 \mu\Omega/\text{cm}$$

$$a = \frac{L}{R}$$

$$= \frac{1.7 \times 300\,000}{10^{-6} \times 14.4}$$

$$= \frac{1.7 \times 3}{144} = \frac{5.1}{144} \text{ cm}^2$$

$$= \frac{51}{1440} \text{ cm}^2 = 0.035 \text{ cm}^2$$

$$\text{Now } a = \frac{\pi d^2}{4} \text{ or } d^2 = \frac{a \times 4}{\pi}$$

$$d = \sqrt{\frac{a \times 4}{\pi}}$$

$$= \sqrt{\frac{0.035 \times 4 \times 7}{22}}$$

$$= \sqrt{0.0445}$$

$$= 0.21 \text{ cm}$$

$$= 2.1 \text{ mm.}$$

Resistors

Objectives: At the end of this lesson you shall be able to

- explain the construction and characteristics of various types of resistors
- explain the functions and applications of the resistors in electrical and electronic circuits.

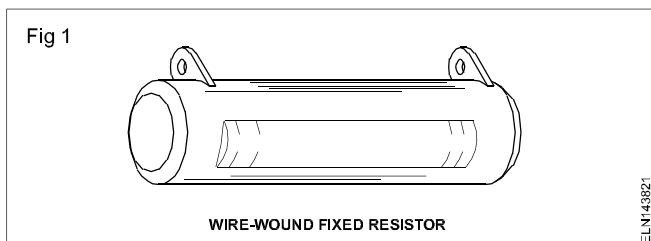
Resistors: These are the most common passive component used in electrical and electronic circuits. A resistor is manufactured with a specific value of ohms (resistance). The purpose of using a resistor in circuit is either to limit the current to a specific value or to provide a desired voltage drop (IR). The power rating of resistors may be from fractional watts to hundreds of Watts.

There are five types of resistors

- 1 Wire-wound resistors
- 2 Carbon composition resistors
- 3 Metal film resistors
- 4 Carbon film resistors
- 5 Special resistors

1 Wire-wound resistors

Wire-wound resistors are manufactured by using resistance wire (nickel-chrome alloy called Nichrome) wrapped around an insulating core, such as ceramic porcelain, bakelite pressed paper etc. Fig 1, shows this type of resistor. The bare wire used in the unit is generally enclosed in insulating material. Wire wound resistors are used for high current application. They are available in wattage ratings from one watt to 100 watts or more. The resistance can be less than 1 ohm and go up to few thousand ohms.

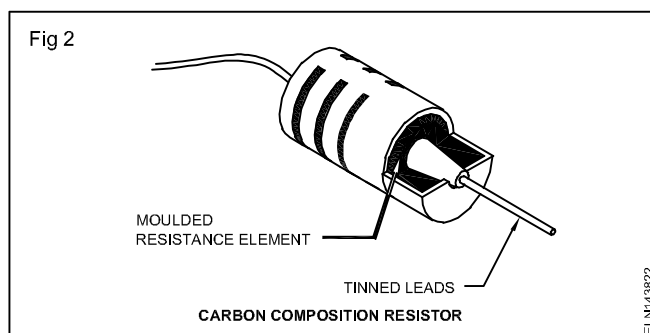


One type of wire-wound resistor is called as fusible resistor enclosed in a porcelain case. This resistor is designed to open the circuit when the current through it exceeds certain limit.

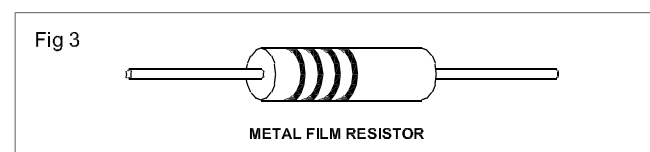
2 Carbon composition resistors

These are made of fine carbon or graphite mixed with powdered insulating material as a binder in the proportion needed for the desired resistance value. Carbon-resistance elements are fixed with metal caps with leads of tinned copper wire for soldering the connection into a circuit. Fig 2 shows the construction of carbon composition resistor.

Carbon resistor are available in values of 1 ohm to 22 megohms and of different power ratings, generally 0.1, 0.125, 0.25, 0.5, 1.0 and 2 watts.



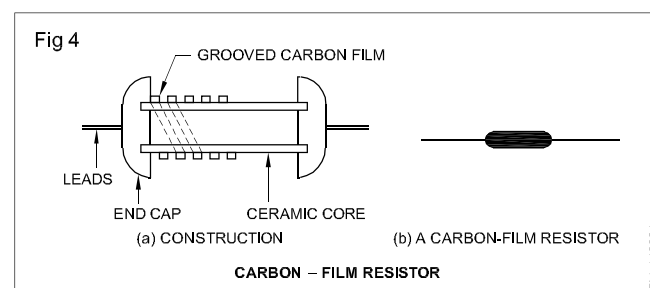
3 Metal film resistors (Fig 3)



Metal film resistors are manufactured by two processes. Thick film resistors are pasted with metal compound and powdered glass which are spread on the ceramic base and then backed.

Thin film resistors are processed by depositing a metal vapour on a ceramic base. Metal film resistors are available from 1 ohm to 10 MΩ, upto 1W. Metal film resistors can work from 120°C to 175°C.

4 Carbon film resistors (Fig 4)



In this type, a thin layer of carbon film is deposited on the ceramic base/tube. A spiral groove is cut over the surface to increase the length of the foil by a specialised process.

Carbon film resistors are available from 1 ohm to 10 meg ohm and up to 1 W and can work from 85°C to 155°C.

All the above four types of resistors are coated with synthetic resin to protect them against mechanical damages and climatic influences, It is therefore, difficult to distinguish them from each other externally.

Specification of resistors : Resistors are specified normally with the four important parameters

- 1 Type of resistor
- 2 Nominal value of the resistors in ohm (or) kilo ohm (or) mega ohm.

3 Tolerance limit for the resistance value in percentage.

4 Loading capacity of the components in wattage

Example

$100 \pm 10\%$, 1W, where as nominal value of resistance is 100Ω .

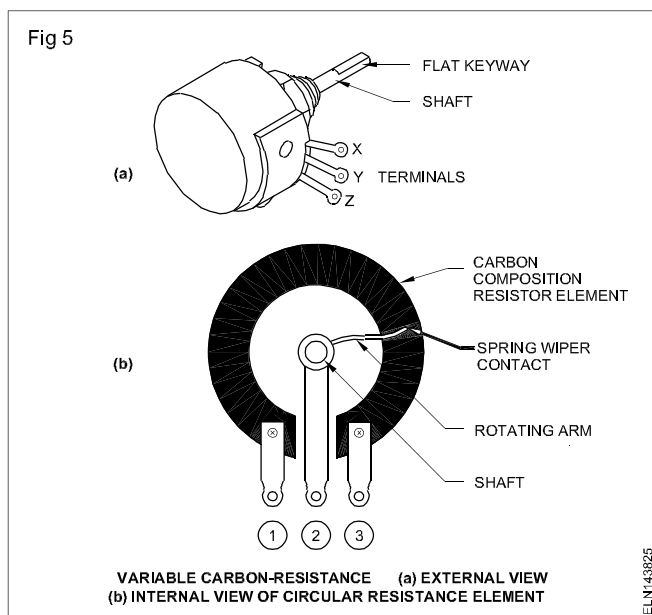
The actual value of resistance may be between 90Ω to 110Ω , and the loading capacity is maximum 1 watt.

The resistors can also be classified with respect to their function as

- 1 Fixed resistors
- 2 Variable resistors

Fixed resistors : The fixed resistors is one in which the is nominal value of resistance is fixed. These resistors are provided with pair of leads. (Fig 1 to 4)

Variable resistors (Fig 5) : Variable resistors are those whose values can be changed. Variable resistors includes those components in which the resistance value can be set at the different levels with the help of sliding contacts. These are known as potentiometer resistors or simply as a potentiometers.



It is provided with 3 terminals as shown in Fig 5 and 6. They are available with carbon tracks (Fig 5) and wire wound (Fig 6) types. Trimmer potentiometers (or) resistor which can be adjusted with the help of a small screw drivers. (Fig 7).

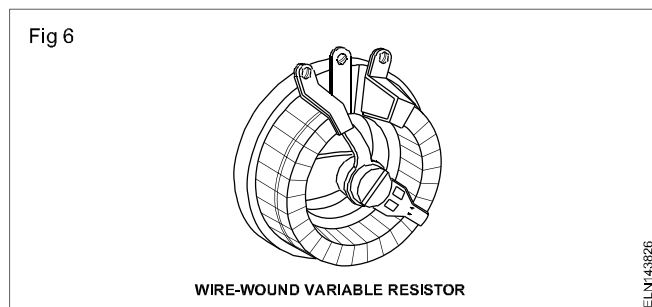
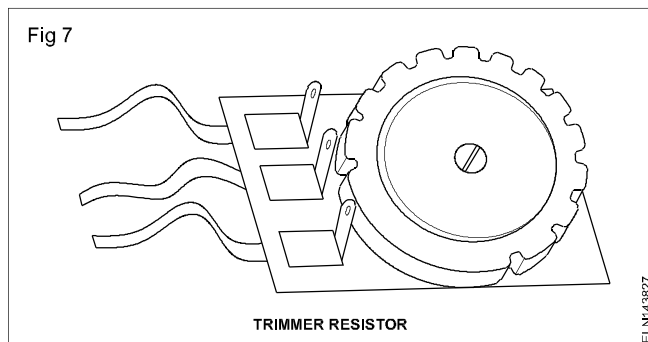


Fig 7



Resistance depends upon temperature, voltage, light : Special resistors are also produced whose resistance varies with temperature, voltage, and light.

PTC resistors (Sensistors) : Since, different materials have different crystal structure, the rate at which resistance increases with raising temperature varies from material to material. In PTC resistor (positive Temp. coefficient resistor), as the temp increases, the resistance increases non linearly. For example, the resistance of PTC at room temperature may be of nominal value 100Ω when the temperature rises say 10°C , it may increase to 150Ω and with further increase of another 10°C , it may increase to 500Ω .

NTC Resistors (Thermistors) : In case of NTC resistors (Negative temperature co-efficient resistors) as the temperature increases, the value of resistance decreases non-linearly, For example, NTC resistor, which has nominal value of resistance is 500Ω at room temperature may decrease to 400Ω with the rise of 10°C temperature and further decrease to 150Ω when the temperature rises to another 10°C .

The PTC and NTC resistors can perform switching operation at specific temperature. They are also used for measurements and temperature compensators.

VDR (Varistors) : The VDR (Voltage dependent resistor) resistance falls non-linearly with increasing voltage. For example, a VDR, may have 100Ω resistance at 10V , and it may decrease to 90Ω at rise in 5V . By further increasing the voltage to another 5V , the resistance may fall to 50Ω . The VDRs are used in voltage stabilisation, arc quenching and over voltage protection.

Light dependent resistor (LDR) : The LDRs are also known as photo-conductors. In LDRs the resistance falls with increase in intensity of illumination. The phenomena is explained as the light energy frees some electron in the materials of the resistors, which are then available as extra conducting electrons. The LDR shall have exposed surface to sense the light. These are used for light barriers in operating relays. These are also used for measuring the intensity of light.

Marking codes for resistors

Objectives: At the end of this lesson you shall be able to

- interpret the coded marking of colours on the resistors
- interpret the letter and digit codes for resistance values
- state the tolerance value for resistors.

Resistance and tolerance value of colour coded resistors

Commercially, the value of resistance and tolerance value are marked over the resistors by colour codes (or) letter and digital codes.

The colour codes for indicating the values to two significant figure and tolerances are given in Table 1 as per IS 8186.

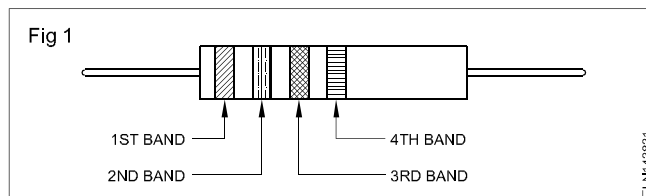
Table 1

Values to two significant figures and tolerances corresponding to colours

Colour	First Band/ Dot	Second Band/ Dot	Third Band/ Dot	Fourth Band/ Dot
	First Figure	Second Figure	Multiplier	Tolerance
Silver	—	—	10^{-2}	$\pm 10\%$
Gold	—	—	10^{-1}	$\pm 5\%$
Black	—	0	1	—
Brown	1	1	10	$\pm 1\%$
Red	2	2	10^2	$\pm 2\%$
Orange	3	3	10^3	—
Yellow	4	4	10^4	—
Green	5	5	10^5	—
Blue	6	6	10^6	—
Violet	7	7	10^7	—
Grey	8	8	10^8	—
White	9	9	10^9	—
None	—	—	—	$\pm 20\%$

The two significant figures and tolerances colour coded resistors have 4 bands of colours coated on the body as in Fig.1.

The first band shall be the one nearest to one end of the component resistor. The second, third and four colour bands are shown in Fig 1.



The first two colour bands indicate the first two digits in the numeric value of resistance. The third colour band indicates the multiplier. The first two digits are multiplied by the multiplier to obtain the actual resistance value. The fourth colour band indicates the tolerance in percentage.

Example

Resistance value : If the colour band on a resistor are in the order- Red, Green, Orange and Gold, then

First colour	Second colour	Third colour	Fourth colour
Red 2	Violet 7	Orange $1000(10^3)$	Gold $\pm 5\%$

the value of the resistor is 27,000 ohms with +5% tolerance.

Tolerance value : The fourth band (tolerance) indicates the resistance range within which is the actual value falls. In the above example, the tolerance is $\pm 5\%$. $\pm 5\%$ of 27000 is 1350 ohms. Therefore, the value of the resistor is any value between 25650 ohms and 28350 ohms. The resistors with lower value of tolerance (precision) are costlier than normal value of resistors.

Methods of measuring low and medium resistance

Objectives: At the end of this lesson you shall be able to

- state the different methods of measuring resistance
- describe the ammeter & voltmeter method.

Classification of resistance: Based on the ohmic value of resistance, we name it as low, medium and high resistance.

A resistance is classified on its ohmic value as low, medium, or high.

Ranges

- Low resistance - one ohm and below one ohm
- Medium resistance - above one ohm up to 100,000 ohms (100 k Ω)
- High resistance - above 100 k Ω (i.e 100000 Ω)

The above classification is not rigid.

Uses

Low resistance: Armature winding, ammeter shunt, cable length, contact resistance.

Medium resistance: All electrical apparatus normally used have resistance in this range - bulbs, heaters, relay, motor starters.

High resistance: Insulation resistance, carbon composition resistors above 100K in the circuit.

We shall limit for the present to the methods used for measuring low and medium resistances in the following section.

Question

- 1 The lamp resistance of a mini-torch light, operating on 1.5 volts is classified as _____ resistance.

Methods of measuring low resistance: The following three methods are used to measure low resistance.

- Voltmeter and ammeter method.
- Comparison of unknown with standard using potentiometer.
- Kelvin bridge
- Shunt type Ohmmeter

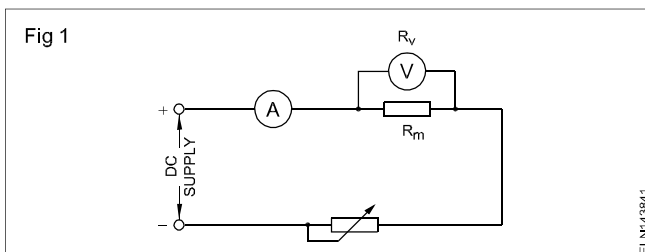
Ammeter and voltmeter method: This method, which is the simplest of all, is very commonly used for the measurement of low resistance.

In Fig 1, R_m is the resistance to be measured and V is a high resistance voltmeter of resistance R_v . A current from a steady direct current supply is passed through R in series

with a suitable ammeter. Then assuming the current through the unknown resistance to be the same as that measured by the ammeter A, the formula is given as

$$R_m = \frac{\text{Voltmeter reading}}{\text{Ammeter reading}}$$

$$R_m = \text{Measured value}$$



If the voltmeter resistance is not very large, compared with the resistor to be measured, the voltmeter current will be an appreciable fraction of the current I , measured by the ammeter, and a serious error may be introduced on this account.

Medium resistance: The following three methods are used to measure medium resistance.

- Series type Ohmmeter
- Voltmeter and ammeter method
- Substitution method
- Wheatstone bridge method

The first method has been considered in the section on low resistance measurement. Substitution method and the wheatstone bridge method is explained subsequently.

Ohmmeter

Objectives: At the end of this lesson you shall be able to

- classify resistances in terms of their values
- explain the principle, construction and use of a series type ohmmeter
- explain the principle, construction and use of a shunt type ohmmeter.

Resistances could be broadly classified according to their values as indicated below.

Low resistance

All resistances of the order of one ohm and below one ohm, may be classified as low resistances.

Example Armature and series field resistances of large DC machines, ammeter shunts, cable resistance, contact resistance etc.

Medium resistances

Resistances above 1 ohm up to 100,000 ohms are classified as medium resistances.

Example Heater resistances, shunt field resistance, relay coil resistance etc.

High resistances

Resistances above 100,000 ohms are classified as high resistances.

Example Insulation resistance of equipment, cables etc.

Measurement of resistances

Medium resistances could be measured by instruments like Kelvin's bridge, Wheatstone bridge, Slide wire bridge, Post Office box and Ohmmeter. Special designs of the above instruments allow measurement of low resistances, accurately.

However, for measuring high resistances, instruments like megohmmeter or megger are used.

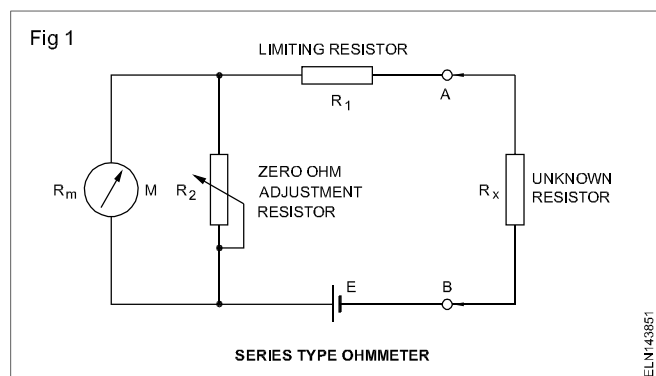
Ohmmeter

The ohmmeter is an instrument that is used for measuring resistance. There are two types of ohmmeters: the series ohmmeter is used for measuring medium resistances and the shunt type ohmmeter is used for measuring low and medium resistances. The ohmmeter in its basic form consists of an internal dry cell, a PMMC meter movement and a current limiting resistance.

Before using an ohmmeter in a circuit, for resistance measurement, the current in the circuit must be switched off and also any electrolytic capacitor in the circuit should be discharged. Remember that the ohmmeter has its own source of supply.

Series type ohmmeter: construction

A series type ohmmeter shown in Fig 1 essentially consists of a PMMC (Permanent magnet moving coil) ('d' Arsonval) movement 'M', a limiting resistance R_1 and a battery 'E' and a pair of terminals A and B to which the unknown resistance ' R_x ' is to be connected. The shunt resistance R_2 connected in parallel to meter 'M' is used for adjusting the zero position of the pointer.



Working

When the terminals A and B are shorted (unknown resistor $R_x = \text{zero}$), maximum current flows in the circuit. The meter is made to read full scale current (I_{fsd}) by adjusting the shunt resistance R_2 . The full scale current position of the pointer is marked zero (0) ohm on the scale.

When the ohmmeter leads (A & B terminals) are open, no current flows through the meter movement. Therefore, the meter does not deflect and the pointer remains in the left hand side of the dial. The left side of the dial is marked as infinity (∞) resistance which means that there is infinite resistance (open circuit) between the test leads.

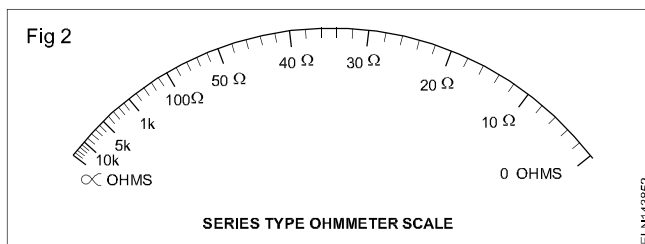
Intermediate marking may be placed in the dial (scale) by connecting different known values of R_x to the instrument terminals A and B.

The accuracy of the ohmmeter depends greatly upon the condition of the battery. The voltage of the internal battery may decrease gradually due to usage or storage time. As such the full scale current drops and the meter does not read zero when the terminals A and B are shorted.

The variable shunt resistor R_2 in Fig 1 provides an adjustment to counteract the effect of reduced battery voltage within certain limits. If the battery voltage falls below a certain

value, adjusting R_2 may not bring the pointer to zero position, and hence, the battery should be replaced with a good one.

As shown in Fig 2, the meter scale will be marked zero ohms at the right end and infinity ohms at the left end.

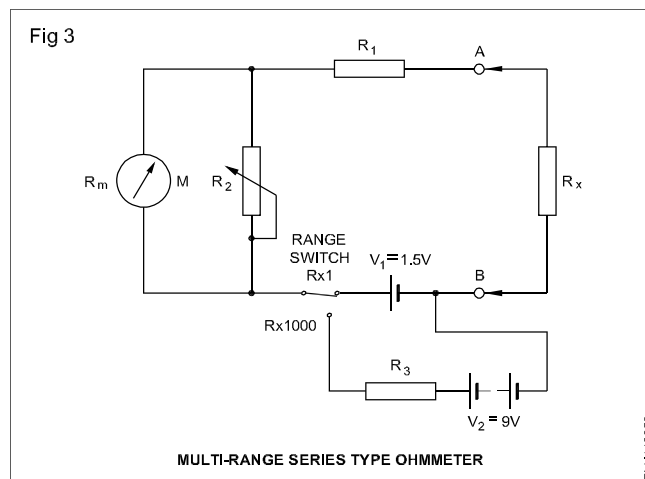


This ohmmeter has a non-linear scale because of the inverse relationship between resistance and current. This results in an expanded scale near the zero end and a crowded scale at the infinity end.

Multiple ohmmeter range

Most of the ohmmeters have a range switch to facilitate measurement of a wide range of resistors, say from 1 ohm up to 100,000 ohms. The range switch acts as the multiplying factor for the ohms scale. To get the actual value of measurement, the scale reading need to be multiplied by the R_x factor of the range switch.

The range switch arrangement is provided either through a network of resistances powered through a cell of 1.5V or through a battery of 9 or 22.5 volts. The arrangement is shown in Fig 3. The resistance value of R_3 is so chosen that the full scale current is passed through the meter at the enhanced source voltage.

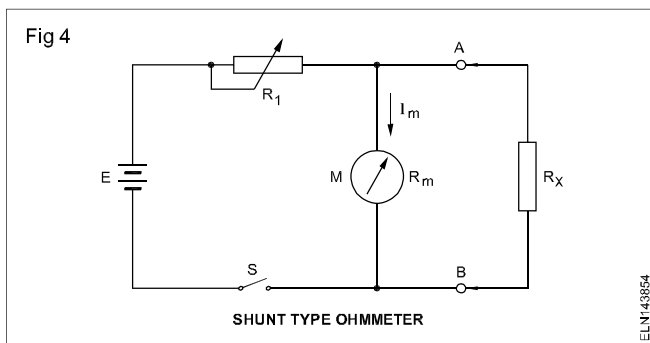


Use

This type of ohmmeter is used for measuring medium resistances only and the accuracy will be poor in the case of very low and very high resistance measurements.

Shunt type ohmmeter

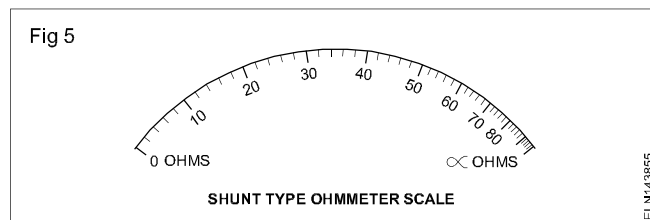
Fig 4 shows the circuit diagram of a shunt type ohmmeter. In this meter the battery 'E' is in series with the zero ohm, adjustment resistor R_1 and the PMMC meter movement. The unknown resistance R_x which is connected across the terminals A and B forms a parallel circuit with the meter. To avoid draining of the battery during storage, the switch S is of a spring-loaded, push-button type.



Working

When the terminals A and B are shorted (the unknown resistance $R_x = \text{zero ohm}$), the meter current is zero. On the other hand if the unknown resistance $R_x = \infty$ (keeping A and B open) the current flows only through the meter, and by proper selection of the value R_1 , the pointer can be made to read its full scale.

The shunt type ohmmeter, therefore, has the zero mark at the left hand side of the scale (no current) and the infinite mark at right hand side of the scale (full scale deflection current) as shown in Fig 5. When measuring the resistance of the intermediate values the current flow divides in a ratio inversely proportional to the meter resistance and the unknown resistance. Accordingly the pointer takes an intermediate position.



Use

This type of ohmmeter is particularly suitable for measuring low value resistors.

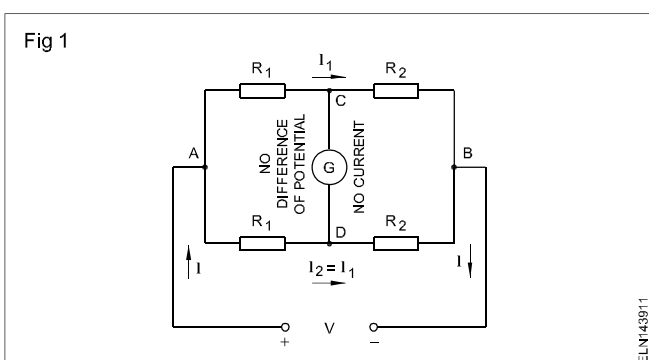
Wheatstone bridge - principle and its application

Objectives: At the end of this lesson you shall be able to

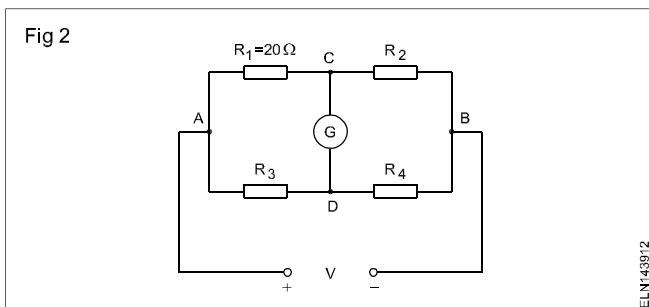
- describe the method of obtaining equal potential points in two branches of a parallel circuit
- state wheatstone bridge circuit, construction, function and uses.
- determine the unknown resistance by the wheatstone bridge.

Points of equal potential in parallel circuits: An electrical current flows only when a potential difference is present. Without a potential difference, current will not flow.

In the Fig 1, the resistances R_1 and R_2 in each of the parallel branches are equal. Therefore, the potential differences across the two resistors R_1 are equal, i.e. from A to C and from A to D. Hence, even when the points C and D are connected with the galvanometer no current will flow.



Four resistors R_1, R_2, R_3 and R_4 are arranged as shown in Fig 2. Select the values of R_2, R_3 and R_4 from the list so that no current flows between points C & D. Resistance values are 20 ohms, 30 ohms, 40 ohms, 70 ohms, 15 ohms.



Equal resistance ratio in parallel circuit: One does not need equal resistances in the parallel circuits to obtain equal potential nodes. It suffices if the resistances are in the same ratio to each other.

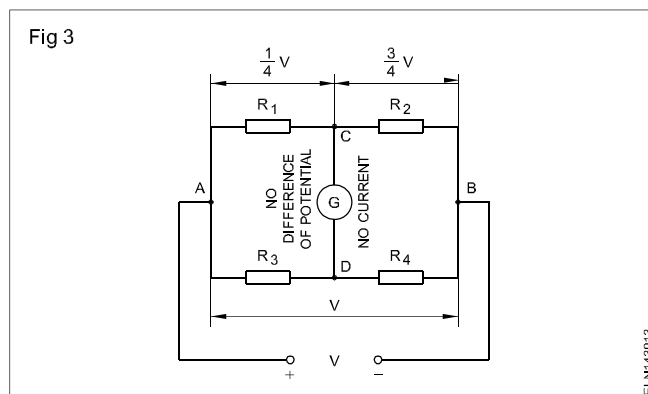
In the circuit diagram (Fig 3), the resistances in the top conductor branch are in the ratio 1 : 3.

The resistances in the bottom conductor branch are also in the ratio 1:3. The supply emf V , is therefore, divided in the both conductor branches in the same ratio 1:3. The first potential difference is

$$= \frac{1}{4} V \text{ and second} = \frac{3}{4} V$$

Again no current can flow in a conductor connected across the points C and D.

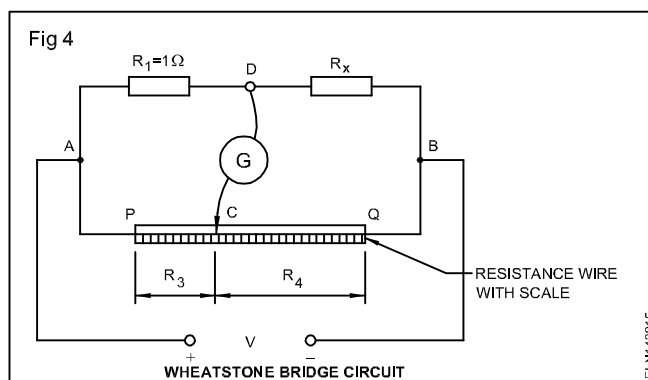
A conductor between C and D is called a bridge connection.



The wheatstone bridge circuit

The equal resistance ratio in parallel circuits can be used for the measurement of resistance.

In the circuit arrangement shown in Fig 4, the sliding contact C slides along a resistance wire.



R_1 is a standard resistor, e.g. 1 ohm.

The sliding contact C is moved along the resistance wire until the detector or bridge galvanometer across C-D reads zero. Then the resistance ratios in the two parallel branches are equal.

$$R_x : R_1 = R_4 : R_3$$

If $R_1 = 1 \text{ ohm}$ then

$$R_x = \frac{R_4}{R_3}$$

This circuit arrangement can, therefore, be used to measure

an unknown resistance R_x . The resistance can be directly read from a scale on the resistance wire. (Fig 4)

For determining the unknown resistance by Wheatstone Bridge

- The current flowing through the bridge connection should be zero.
- The values of the other three resistances should be precisely known.

How to find no current flows through the bridge connection?: An instrument, that can indicate the flow of even a few micro amperes (millionth of an ampere), called galvanometer, is used. There are galvanometers that give full scale deflection for 25 microamperes.

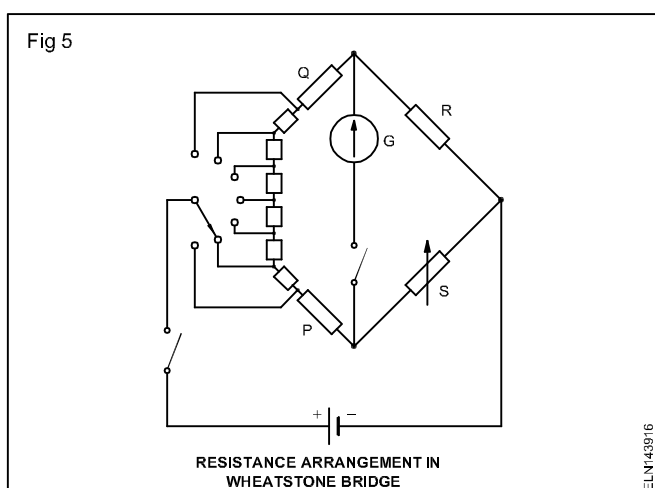
In the professional Wheatstone bridges, the galvanometer is provided with a parallel resistance and switch. The bridge connection is made only by pressing a push button. This enables the user to check a momentary deflection of the meter. In the case of excessive deflection, adjustment of the variable resistor is done. Final and precise adjustment of the variable resistance is made keeping the shunt resistor of the galvanometer open.

The three arms of the bridge are made of standard/precision resistors. The contact resistance is kept very low to increase the accuracy of the measurement made by the Wheatstone bridge.

In short, the use of the galvanometer is to ensure that the current through the bridge connection is zero, i.e. both parallel branches have equipotential points connected by the bridge connector.

This arrangement is named after its inventor and is called the Wheatstone Bridge.

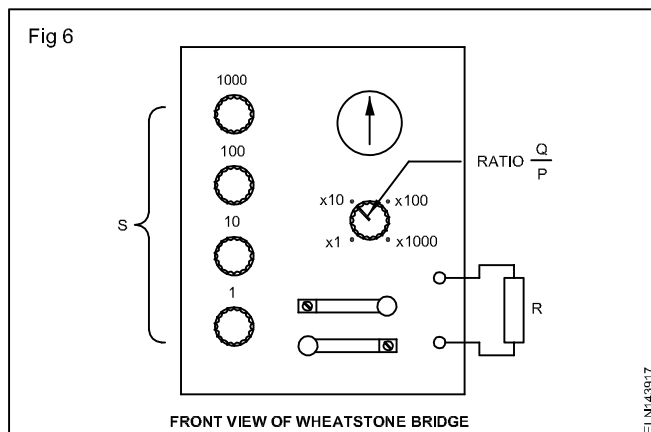
The Wheatstone Bridge is used for measurements in the range of about 1.0 ohm to 1.0 megohm. In Fig 5, resistors P, Q and S are internal to the instrument. R is the resistor of unknown value to be measured.



The instrument is adjusted until the ratio $\frac{Q}{P} = \frac{R}{S}$

This is indicated by a zero reading on the galvanometer with its switch in the closed position.

The resistors P and Q are called ratio arms. P and Q are varied in steps to give a range of values and the resistance value of 'S' is set by the decade resistance S. (Fig 6)

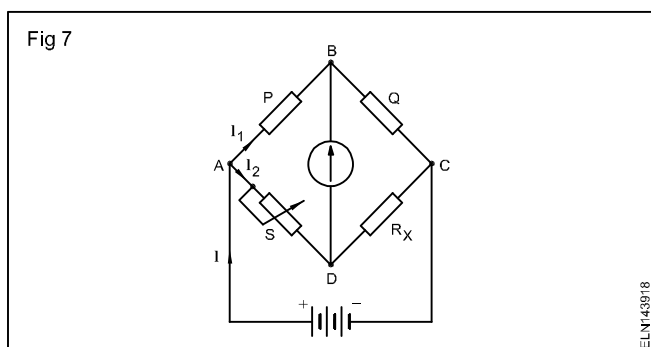


$$R = \frac{Q}{P} \text{ multiplied by } S.$$

The ratio $\frac{Q}{P}$ is arranged to be 1, 10, 100 or 1,000 for ease of calculation.

S is the variable resistance. Four decade resistances are connected in series. The value of S can be set in steps of one ohm from 1.0 ohm to 9999 ohms by suitably setting the four decade resistance units.

Example 1: The Wheatstone Bridge circuit is used to determine the value of the unknown resistor R_x . The bridge is balanced when $P = 100$ ohms, $Q = 1000$ ohms and S is adjusted to 130 ohms. Calculate the value of the unknown resistor R_x . (Fig 7)



Solution

$$\text{At balance } V_{AB} = V_{AD}$$

$$\text{and } V_{BC} = V_{DC}$$

$$\text{therefore, } I_1 P = I_2 S$$

$$\text{and } I_1 Q = I_2 R_x$$

$$\frac{I_1}{I_2} = \frac{S}{P} = \frac{I_1}{I_2} = \frac{R_x}{Q}$$

$$\frac{S}{P} = \frac{R_x}{Q}$$

$$R_x = \frac{S}{P} \times Q = \frac{130 \times 1000}{100}$$

$$R_x = 1300 \, \Omega$$

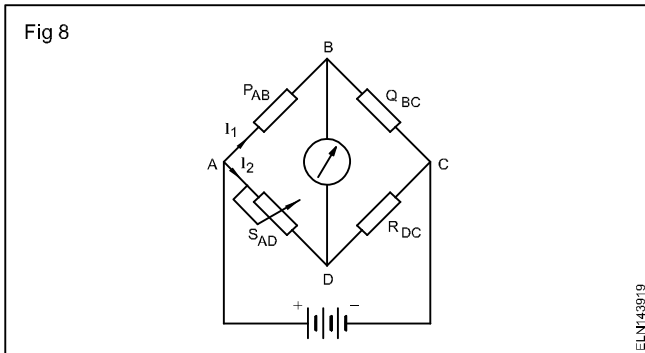
Example 2 : In the Wheatstone Bridge network (Fig 8), ABCD is balanced when

$$P_{AB} = 500 \text{ ohms}$$

$$Q_{BC} = 250 \text{ ohms and}$$

$$S_{AD} = 12 \text{ ohms.}$$

Determine the value of R_{DC} .



Solution

At balance $V_{AB} = V_{AD}$

and $V_{BC} = V_{DC}$

$$I_1 P = I_2 S$$

$$\text{and } I_1 Q = I_2 R$$

Therefore,

$$\frac{I_1}{I_2} = \frac{S}{P} = \frac{I_1}{I_2} = \frac{R}{Q}$$

$$\frac{S}{P} = \frac{R}{Q} \text{ and } R = \frac{S}{P} \times Q$$

$$R = \frac{12}{500} \times 250 = 6 \text{ ohms}$$

Effect of variation of temperature on resistance

Objectives: At the end of this lesson you shall be able to

- explain on what factors electrical resistance of a conductor depends
- state the temperature co-efficient of resistance.

The resistance of material largely depends on temperature and varies according to the material. The phenomenon is used to develop special resistors, PTC & NTC etc., but the overall effect of temperature normally increase the current in that conductor material.

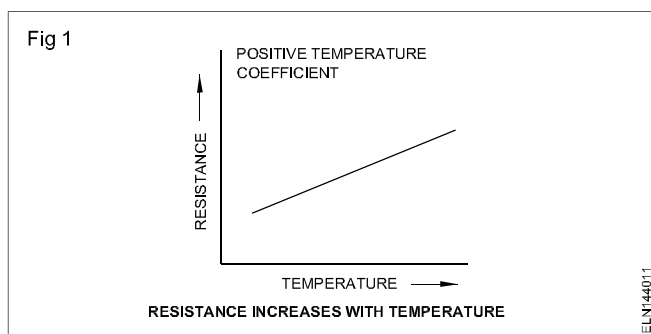
When resistance r is a constant depending on the nature of the material of the conductor and known as its specific resistance or resistivity. Dependency of resistance on temperature is explained in detail below:-

Effect of temperature on resistance: Actually, the relative values of resistance that were given earlier apply to the metals when they are at about room temperature. At higher or lower temperatures, the resistances of all materials change.

In most cases, when the temperature of a material goes up, its resistance goes up too. But with some other materials, increased temperature causes the resistance to go down.

The amount by which the resistance is affected by each degree of temperature change is called the temperature coefficient. And the words positive and negative are used to show whether the resistance goes up or down with the temperature.

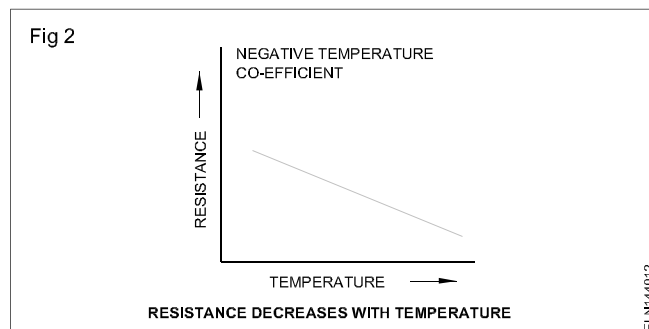
When the resistance of the material goes up as temperature is increased, it has a positive temperature coefficient. It is appropriate in the case of pure metals such as silver, copper, aluminium, brass etc. (Fig 1)



In the case of certain alloys such as eureka, manganin, etc. increase in resistance due to increase in temperature is relatively less and irregular.

When a material's resistance goes down as the temperature is increased, it has a negative temperature coefficient. (Fig 2)

This applies in the case of electrolytes, insulators such as paper, rubber, glass, mica etc. and partial conductors such as carbon.



Temperature coefficient of resistance (a) of a conductor: Let a metallic conductor, having a resistance of R_0 at 0°C , be heated to $t^\circ\text{C}$ and let its resistance at this temperature be R_t . Then, considering normal ranges of temperature, it is found that the increase in resistance depends:

- directly on its initial resistance
- directly on the rise in temperature
- on the nature of the material of the conductor

$$\text{Hence } (R_t - R_0) = R_0 t \alpha \quad \dots(i)$$

where α (alpha) is constant and is known as the temperature coefficient of resistance of the conductor.

Rearranging Eq.(i), we get

$$= \frac{R_t - R_0}{R_0 \times t} = \frac{R}{R_0 \times t}$$

$$\text{If } R_0 = 1\Omega, t = 1^\circ\text{C}, \text{ then } \alpha = \Delta R = R_t - R_0.$$

Hence, the temperature-coefficient of a material may be defined as: the change in resistance in ohm per $^\circ\text{C}$ rise in temperature.

$$\text{From Eq.(i), we find that } R_t = R_0(1 + \alpha t) \quad \dots(ii)$$

In view of the dependence of α on the initial temperature, we may define the temperature coefficient of resistance at a given temperature as the change in resistance per ohm per degree centigrade change in temperature from the given temperature.

In case R_0 is not given, the relationship between the known resistance R_1 at $t_1^\circ\text{C}$ and the unknown resistance R_2 at $t_2^\circ\text{C}$ can be found as follows:

$$R_2 = R_0(1 + \alpha_0 t_2) \text{ and}$$

$$R_1 = R_0(1 + \alpha_0 t_1).$$

Therefore $\frac{R_2}{R_1} = \frac{1 + \alpha_0 t_2}{1 + \alpha_0 t_1}$

Resistivities and temperature coefficients

Material Metals-Alloys	Resistivity in ohm-metre at 20°C $\times 10^{-8}$	Temperature coefficient at 20°C $\times 10^{-4}$
Aluminium	2.8	40.3
Brass	6 – 8	20
Carbon	3000 – 7000	–(5)
Constant or Eureka	49	(+0.160 – 0.4)
Copper (annealed)	1.72	39.3
German silver	20.2	2.7
Iron	9.8	65
Manganin (84% Cu; 25% Mn; 4% Ni)	44 – 48	0.15
Mercury	95.8	8.9
Nichrome (60% Cu; 25% Fe; 15% Cr)	108.5	1.5
Nickel	7.8	54
Platinum	9 – 15.5	36.7
Silver	1.64	38
Tungsten	5.5	47

Insulators	Resistivity in ohm-metre at 20°C	Temperature coefficient at 20°C
Amber	5×10^{14}	10^{12}
Bakelite	10^{10}	
Glass	$10^{10} - 10^{12}$	
Mica	10^{15}	
Rubber	10^{16}	
Shellac	10^{14}	
Sulphur	10^{15}	

Example: The resistance of a field coil measures 55 ohms at 25°C and 65 ohms at 75°C. Find the temperature-coefficient of the conductor at 0°C.

$$R_t = R_0(1 + \alpha_0 t)$$

$$R_{25} = 55 = R_0(1 + 25\alpha_0) \quad \dots \text{Eqn.1}$$

$$R_{75} = 65 = R_0(1 + 75\alpha_0) \quad \dots \text{Eqn.2}$$

Dividing Eqn.2 by Eqn.1 we get

$$\frac{R_{75}}{R_{25}} = \frac{65}{55} = \frac{1 + 75\alpha_0}{1 + 25\alpha_0}$$

$$\frac{13}{11} = \frac{1 + 75\alpha_0}{1 + 25\alpha_0}$$

Cross multiplying we get

$$13[1 + 25\alpha_0] = 11[1 + 75\alpha_0]$$

$$13 + 325\alpha_0 = 11 + 825\alpha_0$$

$$13 - 11 = 825\alpha_0 - 325\alpha_0$$

$$2 = 500\alpha_0$$

$$\alpha_0 = \frac{2}{500} = 0.004 \text{ per } ^\circ\text{C}.$$

Series and parallel combination circuit

Objectives: At the end of this lesson you shall be able to

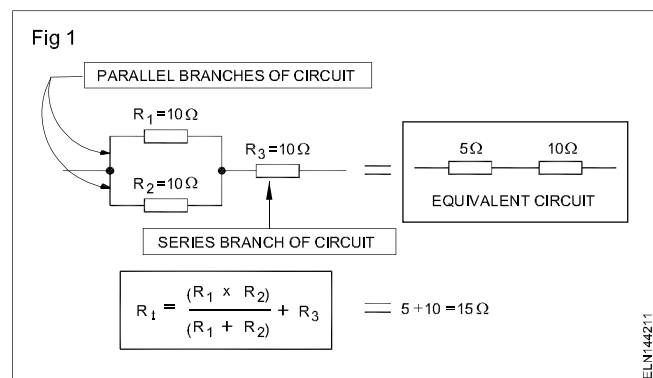
- compare the characteristics of series and parallel circuits
- solve series-parallel circuit problems

Comparison of characteristics of DC series and parallel circuits

Sl. No.	Series circuit	Parallel circuit
1	The sum of voltage drops across the individual resistances equals the applied voltage.	The applied voltage is the same across each branch.
2	The total resistance is equal to the sum of the individual resistances that make up the circuit. $R_t = R_1 + R_2 + R_3 + \dots$ etc combination.	The reciprocal of the total resistance equals the sum of the reciprocal of the resistances. The resultant resistance is less than the smallest resistance of the parallel
3	Current is the same in all parts of the circuit. resistance of each branch.	The current divides in each branch according to the resistance of each branch
4	Total power is equal to the sum of the power dissipated by the individual resistances.	(Same as series circuit) Total power is equal to the sum of the power dissipated by the individual resistances.

Formation of series parallel circuit

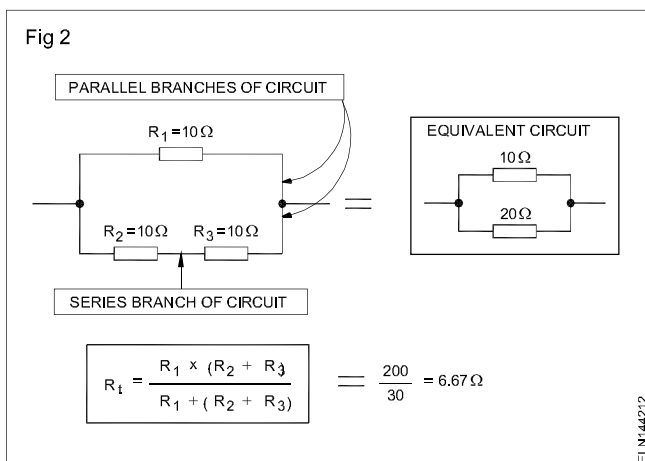
Apart from the series circuit and parallel circuits, the third type of circuit arrangement is the series-parallel circuit. In this circuit, there is at least one resistance connected in series and two connected in parallel. The two basic arrangements of the series-parallel circuit are shown here. In one, resistor R_1 and R_2 are connected in parallel and this parallel connection, in turn, is connected in series with resistance R_3 . (Fig 1)



Thus, R_1 and R_2 form the parallel component, and R_3 the series component of a series-parallel circuit. The total resistance of any series-parallel circuit can be found by merely reducing it into a simple series circuit. For example, the parallel portion of R_1 and R_2 can be reduced to an equivalent 5-ohm resistor (two 10-ohm resistors in parallel).

Then it has an equivalent circuit of a 5-ohm resistor in series with the 10-ohm resistor (R_3), giving a total resistance of 15 ohms for the series-parallel combination.

A second basic series-parallel arrangement is shown in Fig 2 where basically it has two branches of a parallel circuit. However, in one of the branches it has two resistances in series R_2 and R_3 . To find the total resistance of this series-parallel circuit, first combine R_2 and R_3 into an equivalent 20-ohm resistance. The total resistance is then 20 ohms in parallel with 10 ohms, or 6.67 ohms.



Combination circuits

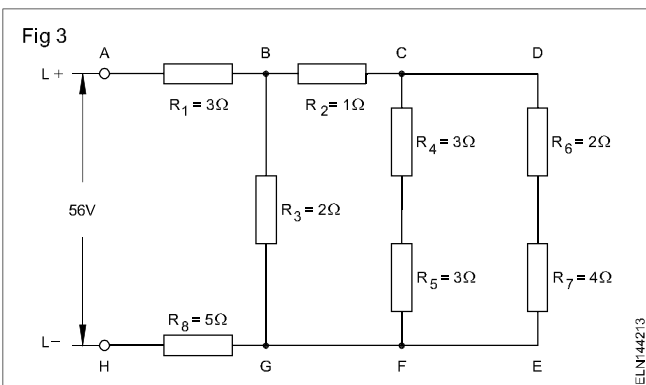
A series-parallel combination appears to be very complex.

However, a simple solution is to break down the circuit into series/or parallel groups, and while solving problems, each may be dealt with individually. Each group may be replaced by one resistance, having the value equal to the sum of all resistances.

Each parallel group may be replaced by one resistance value equivalent to the combined resistance of that group. Equivalent circuits are to be prepared for determining the current, voltage and resistance for each component.

Example

Determine the combined resistance of the circuit shown in Fig 3.



Procedure

- 1 Combine R_6 and R_7 .

$$R_a = R_6 + R_7$$

$$R_a = 2 + 4$$

$$R_a = 6 \text{ ohms.}$$

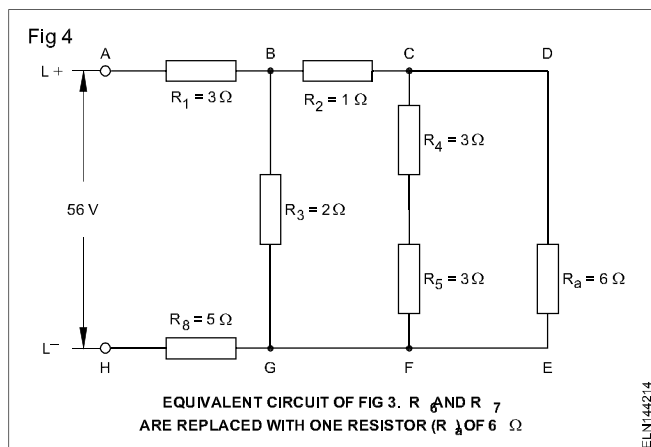
- 2 Draw an equivalent circuit with resistance R_a . (Fig 4)

- 3 Combine R_4 and R_5 of Fig 4.

$$R_b = R_4 + R_5$$

$$R_b = 3 + 3$$

$$R_b = 6 \text{ ohms.}$$

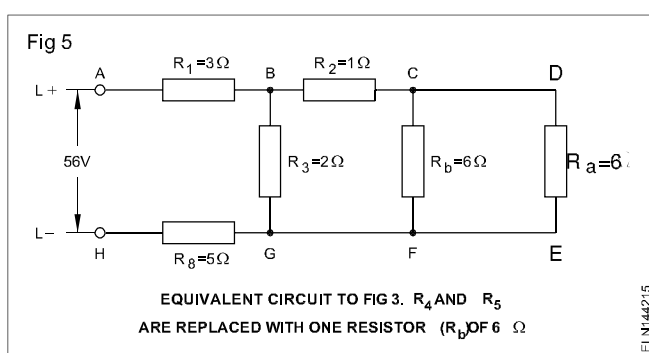


- 4 Draw an equivalent circuit as per Fig 5.

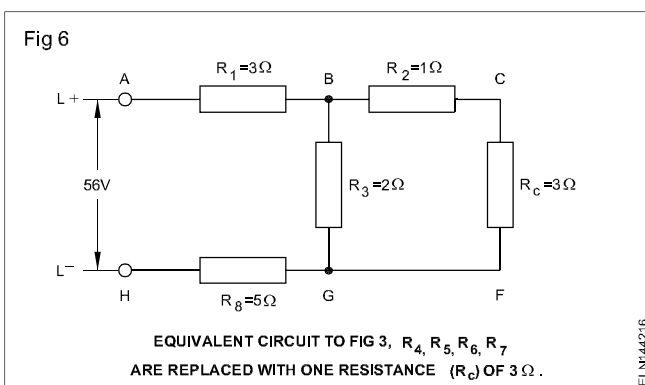
- 5 Combine R_a and R_b and call the equivalent resistance value as R_c . (Fig 5)

$$R_c = \frac{R_a \times R_b}{R_a + R_b} = \frac{6 \times 6}{6 + 6}$$

$$= \frac{36}{12} = 3 \text{ ohms}$$



- 6 Draw the equivalent circuit. (Fig 6)

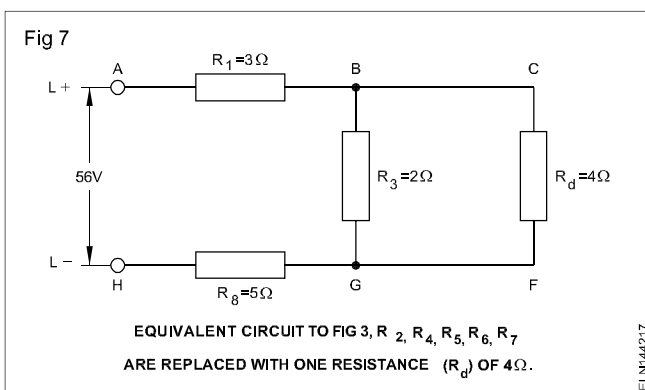


- 7 Combine R_2 and R_c and call the equivalent resistance R_d .

$$R_d = R_2 + R_c$$

$$R_d = 1 + 3 \quad R_d = 4 \text{ ohms.}$$

- 8 Draw an equivalent circuit. (Fig 7)

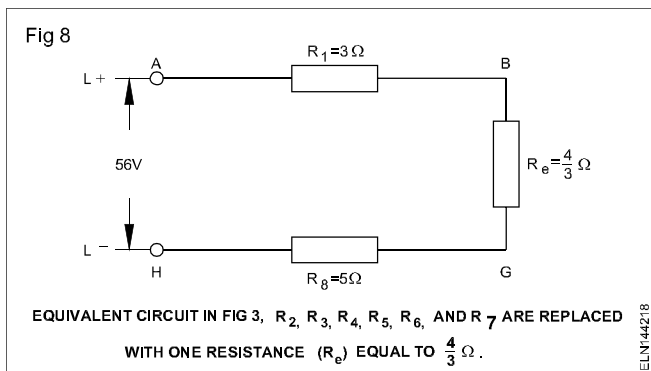


- 9 Now combine R_3 and R_d and call it R_e

$$R_e = \frac{R_3 \times R_d}{R_3 + R_d} = \frac{2 \times 4}{2 + 4}$$

$$= \frac{8}{6} = \frac{4}{3} = 1 \frac{1}{3} \text{ ohms.}$$

10 Draw an equivalent circuit. (Fig 8)



11 Combine R_1 , R_e , and R_8 .

$$R_t = R_1 + R_e + R_8$$

$$R_t = 3 + 1 \frac{1}{3} + 5$$

$$R_t = 9 \frac{1}{3} \text{ ohms.}$$

The total combined resistance of the circuit is $9 \frac{1}{3}$ ohms.

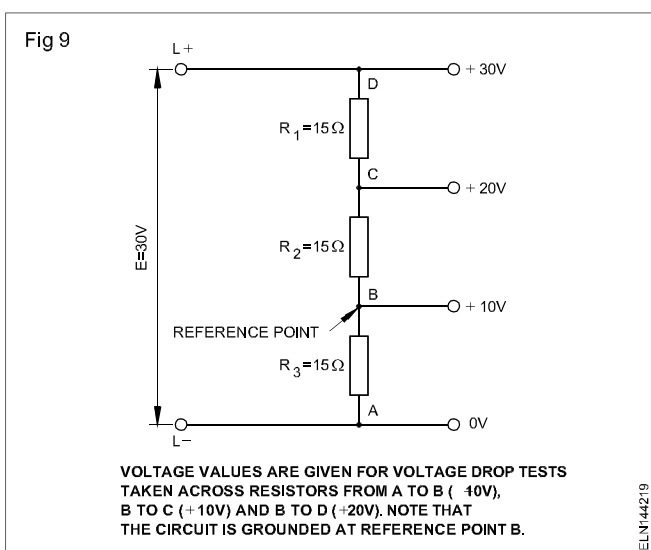
Application

Series-parallel circuits can be used to form a non-standard resistance value which is not available in the market and can be used in the voltage divider circuits.

Voltage divider

If one wants to have different voltages for different parts of a circuit, he can construct a voltage divider. In effect, a voltage divider is nothing more than a series-parallel circuit.

A good voltage divider cannot be designed without first looking at the load resistance. Note in Fig 9 that a voltage divider is made with three 15 ohm resistors to get 10 volts drop across each one.



However, as soon as another resistor (load) is added as in Fig 10, there is a further change. The load resistor serves to drop the total resistance of the lower part of the voltage divider. Use this formula for finding the equivalent resistance (R_{eq}) of resistors of equal value in a parallel circuit:

$$R_{eq} = \frac{r}{N}$$

$$R_{eq} = \frac{15}{2} = 7.5 \text{ ohms.}$$

The equivalent resistance of these two 15 ohm resistors in the lower part of the voltage divider is 7.5 ohms. What will happen to the current and voltage in the circuit as a result of this resistance change?

Remember that, as resistance goes down, current goes up. Therefore, with the addition of the load resistor, the circuit will now carry higher amperage but the voltage between points A and B as well as A and C changes. It is important, then, when constructing a voltage divider circuit, to watch the resistance values which change both voltage and current values. Study Fig 10 carefully to make sure you understand how a voltage divider works.

